

Self-Organized Origami

L. Mahadevan^{1*} and S. Rica²

The controlled folding and unfolding of maps, space structures, wings, leaves, petals, and other foldable laminae is potentially complicated by the independence of individual folds; as their number increases, there is a combinatorial explosion in the number of folded possibilities. The artificially constructed Miura-ori (*I*) pattern, with a periodic array of geometrically and elastically coupled mountain and valley folds (Fig. 1A), circumvents this complication by allowing the entire structure to be folded or unfolded simultaneously. Making such a pattern is not easy, so it may be surprising to find an elegant natural counterpart that is a few hundred millennia old. In Fig. 1B, we show the different stages of the opening of a hornbeam leaf that starts life in its bud as a Miura-ori folded pattern (2). Similar structures arise in insect wings (3) and elsewhere in nature (4), suggesting that these origami patterns are a result of convergent design. This raises a question of mechanism: How might this spatial organization of folds be brought about?

In Fig. 1C, we show the realization of a simple physical solution to this question. The biaxial compression of a thin, stiff, elastic film (with Young's modulus E , Poisson ratio ν , thickness h , and size $L \gg h$) supported on a thick, soft substrate (with Young's modulus $E_p \ll E$ and thickness $H \gg h$) yields into a Miura-ori pattern without any external guidance other than that induced by relatively benign, isotropic, compressive strains that arise because of the relative expansion and contraction between the film and substrate induced by thermal (5) or desiccating (6) effects. Initially, we get primary buckles with wavelength $\lambda \sim h(E/E_p)^{1/3}$ (5), which is very small compared to the lateral extent of the system. However, at the onset of the instability, these straight primary buckles do not have any preferred orientation in a large system and instead form large uncorrelated patches. Non-linear deformations of these primary buckles, through global compression or extension parallel or perpendicular to their orientation, lead to modulational instabilities wherein the buckles collectively deform through soft modes, which are energetically cheaper than the local extension or compression of individual buckles (supporting online text). Thus, the Miura-ori pattern is just the natural response of a softly

supported stiff skin to weak compression along the primary buckles (or weak extension perpendicular to them), wherein the buckles tilt into a zigzag pattern separated by kinks.

Quantifying this through a mathematical analysis of the equations of elasticity (supporting online text) away from the onset of the instability leads to the Newell-Whitehead-Segel equation (7, 8) for the complex-valued amplitude $A(x,y)$

$$\epsilon A + \frac{h^2}{12(1-\nu^2)} \times \left(\partial_x - \frac{i}{2k_c} \partial_{yy} \right)^2 A - g|A|^2 A = 0 \quad (1)$$

Here $Re[A(x,y)e^{ik_c x}]$ is the vertical deflection of the skin, $k_c = 2\pi/\lambda$ is the wave number at onset, ϵ characterizes the distance from the instability threshold, and g characterizes the saturation amplitude. The form of Eq. 1 follows from symmetry considerations (supporting online text) and describes a variety of planform patterns, including the zigzag patterns found in fluid convection, superconductivity, liquid crystals, etc. Our interpretation in the context of folding patterns suggests that Eq. 1 also provides a

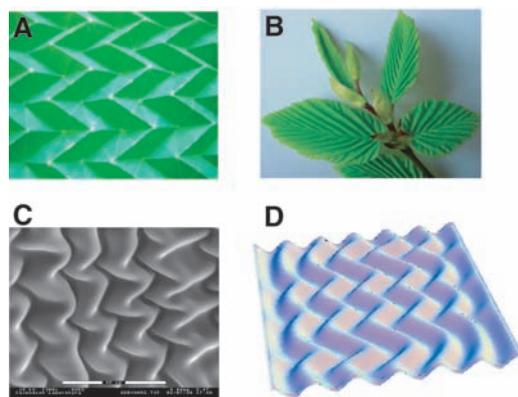


Fig. 1. (A) Plan view of a paper Miura-ori pattern (size, 5 cm), showing the periodic mountain-valley folds. The sharp re-entrant creases that come together at kinks allow the whole structure to fold or unfold simultaneously. (B) Hornbeam leaves (length, 5 cm) in the process of blooming show a natural occurrence of Miura-ori. A single row of kinks along the midrib allows a folded leaf to be deployed once the bud opens (2), as seen in the different stages of leaf opening (clockwise from the top). (C) Zigzag Miura-ori patterns in a thin film atop a thick elastic substrate that is compressed biaxially manifest here in a drying slab of gelatin with a thin skin that forms naturally (6), showing the physically driven self-organization of Miura-ori. Scale bar, 35 μm . (D) Simulations of Eq. 1 yield Miura-ori patterns that arise as a modulational instability of the primary (straight) wrinkles (supporting online text).

natural mathematical framework for the self-organization of Miura-ori. Indeed, a numerical simulation of Eq. 1 in a rectangular domain with periodic boundary conditions in one direction and Neumann conditions in an orthogonal direction reproduces the Miura-ori patterns with creases of wavelength λ (Fig. 1D).

Although Eq. (1) is asymptotically valid only in the weakly nonlinear regime, in practice it describes the patterns well even far from the onset of the zigzag folds. Additionally, the strong localization of the creases and kinks follows naturally from the nonlinear evolution of the pattern in light of the small thickness of the skin-like upper film and the softness of the substrate, leading to almost isometric mountain-valley fold patterns (Fig. 1, A and C). The size d of the kinks is determined by minimizing the sum of the kink-bending energy $U_k \sim Eh^3 \ln(R/d)$, due primarily to conical bending of the thin sheet of size R , and the additional energy of deforming the attached substrate below the kinks, $U_s \sim E_p d^3$. This yields $d \sim h(E/E_p)^{1/3} \sim \lambda$, consistent with observations (Fig. 1C).

Our observations and analysis provide a mechanism for naturally occurring Miura-ori. Stresses induced by the relative growth of stiff skins on soft supports will spontaneously fold into structures such as those shown in Fig. 1; stress-mediated apoptosis may then separate the skin from the tissue to form deployable laminae such as leaves and insect wings.

References and Notes

1. K. Miura, *Proceedings of the 31st Congress of the International Astronautical Federation, IAF-80-A 31*, (American Institute for Aeronautics and Astronautics, New York, 1980), pp. 1–10.
2. H. Kobayashi, B. Kresling, J. Vincent, *Proc. R. Soc. London Ser. B*, **265**, 147 (1998).
3. F. Haas, R. W. Wootton, *Proc. R. Soc. London Ser. B*, **263**, 1651 (1996).
4. B. Kresling, *Biomimetics* **3**, 105 (1991).
5. N. Bowden, S. Brittain, A. G. Evans, J. Hutchinson, G. Whitesides, *Nature* **393**, 146 (1998).
6. R. Rizzieri, personal communication.
7. L. A. Segel, *J. Fluid Mech.* **38**, 203 (1969).
8. A. C. Newell, J. Whitehead, *J. Fluid Mech.* **38**, 279 (1969).
9. L.M. acknowledges support from the Harvard Materials Research Science and Engineering Center and the Office of Naval Research Young Investigator Program; S.R. acknowledges support from Fondo de Ciencia y Tecnología (FONDECYT), Chile.

Supporting Online Material

www.sciencemag.org/cgi/content/full/307/5716/1740/DC1

SOM Text

Fig. S1

References and Notes

13 September 2004; accepted 2 February 2005
10.1126/science.1105169

¹Division of Engineering and Applied Sciences and Department of Organismic and Evolutionary Biology, Harvard University, Cambridge, MA 02138, USA. ²Departamento de Física, Universidad de Chile, Blanco Encalada 2008, Santiago, Chile.

*To whom correspondence should be addressed.
E-mail: lm@deas.harvard.edu